

11-1 How to Integrate Certain
Rational Functions

Learning Objectives:

$$\int \frac{dx}{(ax + b)^n}$$

$$\int \frac{dx}{ax^2 + bx + c}$$

$$\int \frac{x dx}{ax^2 + bx + c}$$

$$\text{Ex1. } \int \frac{dx}{(3x+2)^5} \quad u = 3x+2 \\ dx = \frac{du}{3}$$

$$\int \frac{1}{3} \cdot u^{-5} du$$

$$\frac{1}{3} \cdot -\frac{1}{4} u^{-4} + C$$

$$-\frac{1}{12} (3x+2)^{-4} + C \quad \frac{-1}{12(3x+2)^4} + C$$

$$\text{Ex2. } \int \frac{dx}{4x^2 + 9}$$

$$\int \frac{\frac{3}{2} du}{9u^2 + 9}$$

$$\frac{1}{2} \cdot \frac{1}{3} \int \frac{du}{u^2 + 1}$$

$$\frac{1}{6} \tan^{-1} u + C$$

$$\frac{1}{6} \tan^{-1} \left(\frac{2}{3}x \right) + C$$

$$\sqrt{4x^2} = \sqrt{9u^2}$$

$$2x = 3u \quad x = \frac{3}{2}u$$

$$u = \frac{2}{3}x$$

$$\frac{du}{dx} = \frac{2}{3}$$

$$du = \frac{2}{3} dx \quad dx = \frac{3}{2} du$$

$$\text{Ex3. } \int \frac{dx}{x^2 + 4x + 13}$$

$$\int \frac{dx}{(x^2 + 4x + \underline{4}) + 13 - 4}$$

$$\int \frac{dx}{\boxed{(x+2)^2} + 9} \Rightarrow \int \frac{dx}{x^2 + 1}$$

$$(x+2)^2 = 9u^2$$

$$x+2 = 3u$$

$$u = \frac{x+2}{3}$$

$$dx = 3 du$$

$$\int \frac{3 du}{9u^2 + 9}$$

$$3 \cdot \frac{1}{9} \int \frac{du}{u^2 + 1}$$

$$\frac{1}{3} \tan^{-1}(u)$$

$$\frac{1}{3} \tan^{-1}\left(\frac{x+2}{3}\right) + C$$

$$\text{Ex4. } \int \frac{dx}{4x^2 + 8x + 13}$$

$$\int \frac{dx}{(4x^2 + 8x) + 13}$$

$$\int \frac{dx}{4(x^2 + 2x + 1) + 13 - 4}$$

$$\int \frac{dx}{4(x+1)^2 + 9}$$

$$4(x+1)^2 = 9u^2$$

$$2(x+1) = 3u$$

$$u = \frac{2}{3}(x+1)$$

$$dx = \frac{3}{2} du$$

$$\int \frac{\frac{3}{2} du}{9u^2 + 9}$$

$$\frac{3}{2} \cdot \frac{1}{9} \int \frac{du}{u^2 + 1}$$

$$\frac{1}{6} \tan^{-1} u = \frac{1}{6} \tan^{-1} \left(\frac{2}{3}(x+1) \right) + C$$

Homework

Integration of Rational Functions
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